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Liquid-metal magnetohydrodynamics with strong magnetic fields: a report on Euromech 70

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This paper is a summary of the first Euromech Colloquium to be held on Magnetohydrodynamics (MHD). It was organized in conjunction with the Centre National de la Recherche Scientifique and held at Grenoble from 16–19 March 1976 with 60 participants from 10 countries present. Papers were presented on laminar and turbulent MHD duct flows; heat transfer and two-phase flows in MHD; the effects of magnetic fields on instabilities and turbulence; the motion of and forces on solid objects in MHD flows; flow-measurement methods, and applications of MHD in the metallurgical industries, in sodium technology and in liquid-metal power generation. Our main conclusion is that there are many industrial applications of the existing body of research findings in MHD, but that quite new research problems have arisen as a result of the new applications, and that these need investigation. MHD lives!

1. Introduction

'Où va la MHD?' was the opening question put to the 70th Euromech Colloquium held at Grenoble from 16–19 March 1976. This was the first Colloquium to be held on magnetohydrodynamics (MHD) and the first to be held in conjunction with the C.N.R.S. (Centre National de la Recherche Scientifique). The special reasons for holding a colloquium on MHD at this time were:

(i) In Europe and the Soviet Union there is a steady and growing interest in practical and experimental studies of liquid-metal MHD. It seemed desirable to bring together the engineers and experimentalists involved to compare techniques, to discuss exciting and new applications of liquid-metal MHD and to consider the new experiments that need doing. Only a special meeting could bring together engineers applying their studies to quite different technological problems. It was a pity that nobody from Riga, Latvia was able to attend, Riga being the largest centre of liquid MHD research.

(ii) Theoretical work on liquid-metal MHD flow problems probably needs a new sense of direction and revitalizing. Compared with the large numbers of research workers engaged in theoretical studies of MHD flows in astrophysics and geophysics (see Moffatt 1973), there has been a steady decline in the research work on the theory of MHD flows of laboratory or industrial scale. As readers of this journal will remember, this contrasts sharply with the period 1956 to 1970, when many theoretical studies of MHD were published. Perhaps it was because only a few of the theories had been or were capable of being tested experimentally, or because no obvious practical application of these studies was evident, that the study of this kind of MHD theory lost its fashionable appeal. It was hoped that at this Colloquium recent practical and experimental investigations could show where theory was needed, and whether previous untested theories were correct or needed revision, and also whether there was much application of all the previous theory.

The aspects of MHD covered in the Colloquium were all concerned with electrically conducting liquids in the presence of strong magnetic fields on laboratory or industrial scales. Thus we excluded some important applications of MHD such as electromagnetic flow meters designed for poorly conducting liquids, at one end, and the geophysical and astrophysical applications at the other. Plasma physics was also excluded. However, within this restricted parameter range there are many interesting and practically important flows. The various categories of flow discussed at this Colloquium were (i) MHD duct flows: fully developed flows in constant-area ducts with uniform magnetic fields; developing flows in varying-area ducts and in non-uniform magnetic fields; secondary flows driven by centrifugal and electromagnetic effects; compression waves in ducts; turbulent flows, (ii) heat transfer and two-phase flows, (iii) instabilities and turbulence, (iv) the effects of electromagnetic forces on solid objects in conducting liquids and applications to the removal of particles of slag from molten metal, (v) techniques for measuring velocity, pressure and electric fields in liquid metals with strong magnetic fields, (vi) electromagnetic flow meters in liquid metals, (vii) applications of liquid-metal MHD in the metallurgical industry, nuclear-reactor sodium cooling systems, energy storage and electrical power generation.

The essential non-dimensional numbers describing these flows are the Reynolds number $Re = u_0 L\rho/\eta$ ($\simeq$ inertia forces/viscous forces), the Hartmann number $M = B_0 L(\sigma/\eta)^{\frac{1}{2}}$ ($\simeq$ electromagnetic forces/viscous forces), the interaction parameter $N = \sigma B_0^2 L/\rho u_0 = M^2/Re$ ($=$ electromagnetic/inertia forces), and the magnetic Reynolds number $R_m = \mu\sigma u_0 L$ ($\simeq$ convected magnetic field/diffused magnetic field). If $R_m < 1$, $R_m \simeq$ induced magnetic field/imposed magnetic field. The definitions and typical values of the parameters in industrial or laboratory applications are

u_0 = velocity (10^{-2}–10 m/s),
L = length (10^{-2}–10 m),
ρ = density (10^3–$1{\cdot}4\times10^4$ kg/m^3),
σ = electrical conductivity (10^5–10^7 mho/m),
B_0 = magnetic flux density ($0{\cdot}1$–10 T),
η = viscosity (10^{-4}–10^{-2} kg/(ms)),
μ = magnetic permeability ($4\pi\times10^{-7}$ H/m).

Thus $10 \lesssim Re \lesssim 10^5$, $10 \lesssim M \lesssim 10^4$, $1 \leqslant N \leqslant 10^4$ and $R_m < 20$.

A recent bibliography on MHD has been published by Energy Research and Development Agency, Washington, D.C., and a detailed survey by Lielausis (1975).

2. MHD duct flow

2.1. *Laminar fully developed flows*

Chabrerie & *Tabeling*† (Laboratoire de Génie Electrique des Universités Paris vi and xi) presented solutions for laminar fully developed flows in rectangular ducts with conducting walls parallel to and non-conducting walls perpendicular to the magnetic field $\mathbf{B}_0$. Since the calculations are to be applied to flow in an annular cavity between a rotating Faraday disk (liquid-metal brushes for a homopolar generator) and an outer electrode (see Chabrerie, Fournet & Maillefert 1976), the conducting walls are assumed to move relative to each other, and currents are applied between them. Fourier series were used, requiring n simultaneous equations to be solved numerically. The computations showed good, better than 10% agreement with the asymptotic theory for fixed walls (Hunt & Stewartson 1965) for $M \geqslant 10$. Whether or not there are moving walls, the central inviscid 'core' flow remains uniform and large velocity changes are found in the boundary layers on all the walls. In discussion it was suggested that other approximate methods may be more efficient for $M < 10$, for example, the variational (Wenger 1970) or Galerkin methods.

Temperley (Edinburgh University) began by describing 2 theoretical methods for analysing fully developed laminar flows in rectangular ducts with very strong transverse magnetic fields (so that $M \gg 1$) and with non-conducting walls AA parallel to and thin walls with variable conductivity BB normal to the magnetic field (figure 1). In the first method the flow is analysed by dividing it up into various asymptotic regions, which amount to a core region, Hartmann layers of thickness $O(M^{-1})$ on the walls BB, side-wall boundary layers of thickness $O(M^{-\frac{1}{2}})$ on the walls AA, and 2 corner regions where these two boundary layers meet each other. Different asymptotic expansions are used depending on the conductivity of the walls BB. The interesting physical results obtained by this method (e.g. the occurrence in the boundary layers of velocities $O(M)$ times the core velocities, and reverse flows) agree with those obtained from the asymptotic limit of exact Fourier-series solutions (Temperley & Todd 1971; Hunt 1965). Separate expansions are required for the corner regions. To obtain higher-order terms in an asymptotic series in powers of $M^{-\frac{1}{2}}$ a principle of minimum singularity has to be invoked (see Cook, Ludford & Walker 1972). These two separate expansions for the side-wall boundary layer and the corner regions are not found to be necessary nor is any invoking of the principle of minimum singularity necessary if integral transforms are used directly on the governing equations near the side walls AA. It was suggested in discussion that if there is a problem of uniqueness it might be more physical to rely on the proven fact that for such flows there is a minimum dissipation theorem, as pointed out by Moffatt (1964).

Hervé & Poirier (Institut d'Etudes Supérieurs Scientifiques, Ile de la Reunion)

† The name of the author who gave the paper is written in italics. Titles of papers are listed with the references.

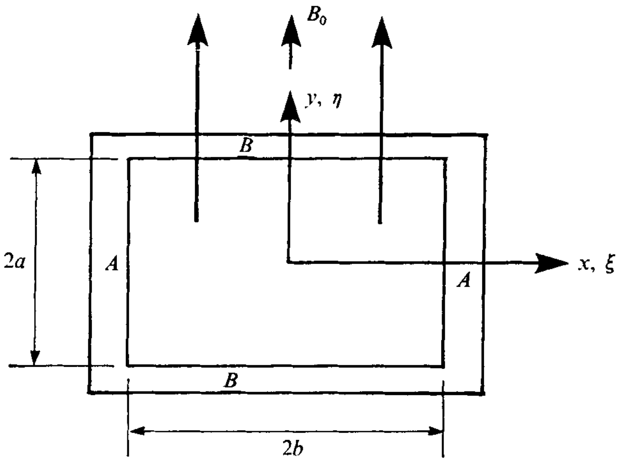

FIGURE 1. Sketch showing the notation for a rectangular duct in a transverse magnetic field.

described another type of laminar flow in a rectangular duct with a moving wall. In the first case the walls BB perpendicular to the magnetic field are highly conducting; one moves and one is stationary; the walls AA are stationary and non-conducting. Straightforward Fourier-series analysis enables the boundary conditions to be satisfied. As with the pressure-driven flows in this duct (Hunt 1965), velocity excesses are found in the side-wall boundary layers, but apparently are only $O(1)$. No asymptotic analysis was performed but flows were computed up to $M = 200$. In the second case one of the walls BB is made non-conducting and this one moves. The core flow is reduced to almost zero. To test the theory experiments had been performed in 2 small rigs in which mercury flows in rectangular annuli ($1 \text{ cm} \times 1 \text{ cm}^2$) and ($4 \text{ cm} \times 4 \text{ cm}$) were driven in one case by a disk rotating above a fixed channel and in the second case by the channel rotating below a fixed disk. The magnetic field was axial and wall conductivity was as in the theory. The mean radii of the annuli were 5 cm and 17 cm and $M < 100$. Flows, measured with electrical potential and Pitot tubes, agreed well with the theory, implying, presumably, that the magnetic field was strong enough for secondary flows to be negligible.

In discussion Shercliff commented that where the field is uniform, M is large and the flows are laminar, it is possible to predict the core flow and the form of the shear layers for rectilinear flows with almost any geometry, any distribution of non-conducting or highly conducting surfaces and current distribution. The rules based on the solutions of the differential equations were set out by Hunt & Shercliff (1971), partly using the ideas of Kulikovskii (1968). The results of Hervé & Poirier and Chabrerie & Tabeling were quite consistent with the rules. There are only one or two exceptional cases which do not follow the general rules (Shercliff 1975), and more investigation into the reasons for this seems indicated.

2.2. *Developing laminar flows*

Otte (Tech. Univ. Berlin) described his numerical calculations of laminar duct flow in a constant-area channel in strong non-uniform transverse magnetic fields at finite magnetic Reynolds number R_m. The flows were assumed to be two-dimensional in the sense that the velocity and magnetic field vectors were $\mathbf{u} = (u, v, 0)$, and $\mathbf{B} = (B_x, B_y, 0)$. Throughout the flow $\partial/\partial z = 0$. The alternating-direction method was used for the nonlinear equations for vorticity and vector potential, and the fast-Fourier-transform (in the x direction) method for calculating the stream function from the vorticity. The most interesting calculation was for a flow where the magnetic field B_y was applied for a finite length of the duct, $N \simeq 2800$ and $R_m = 10$. This corresponds to the liquid-metal MHD experiment at the Technical University in Berlin. Assuming a symmetric return current path, the magnetic field was swept downstream and jet-like (or 'M') velocity profiles were found *in the plane* of the magnetic field (i.e. in the plane normal to that in which they are observed when $R_m \ll 1$ and the duct walls are non-conducting).

In discussion it was pointed out that the analysis would only be applicable to a real duct if the side walls AA were highly conducting so that the electric field in the z direction was constant. By comparing Otte's results for $N \gg 1$ and $R_m > 1$ with the analysis of Holroyd (1976) for a similar two-dimensional, non-uniform duct flow when $N \gg 1$ and $R_m \ll 1$, it can be seen that the effect of raising R_m seems to be to accentuate the tendency of a non-uniform field to produce non-uniform velocity profiles.

Holroyd & *Hunt* (Cambridge) gave a brief review of laminar MHD flows in ducts with changing cross-sectional areas and non-uniform magnetic fields. One of the practical applications of the study of MHD duct flows with very strong magnetic fields is in the design of liquid lithium cooling circuits for putative nuclear-fusion reactors, where $Re \sim 3\times 10^4$, and $R_m < 1$. The magnetic field may be so large ($B_0 < 10T$) that $M \simeq 3\times 10^4$, and $N \simeq 3\times 10^4$ (Hunt & Hancox 1971). The walls of the pipes in such a reactor would have to be metal (insulating materials would not last), with conductivity σ_w, but made as thin as possible (thickness t), to reduce the recirculating electric currents and thus reduce the pressure (for structural reasons) and the pressure gradient (to minimize pumping losses). The dimensionless parameter which described the ratio of the electric conductance of the walls to the fluid is $\phi = \sigma_w t/\sigma L$.

Where $\phi \ll 1$, $N \gg 1$, $M \gg 1$, and $R_m \ll 1$, recent work by Kulikovskii (1968, 1973) and Holroyd (1976) shows that in an MHD duct flow where the cross-sectional area or the magnetic field changes, the flow has to travel along *characteristic surfaces* defined by the integral $\mathrm{I} = \int |\mathbf{B}(s)|^{-1} ds$ remaining constant, the integral being taken along magnetic field lines between their points of intersection with the duct. For a constant magnetic field this result reduces to that of Hunt & Ludford (1968). This is a powerful result which enables the general flow pattern to be predicted immediately, for a wide variety of flows. Typically, stagnant regions are found in the core and jet-like flows near the walls, in planes perpendicular to $\mathbf{B}_0$. However, the detailed flow distributions and the additional pressure gradients created by changes in area or $\mathbf{B}$ can only be calculated by taking the next term in the expansion in powers of $M^{-\frac{1}{2}}$ for a non-conducting duct or $\phi^{\frac{1}{2}}$ for a finitely conducting duct (Walker & Ludford 1974*a*, *b*; Holroyd 1976). If the duct walls are highly conducting or the duct is rectangular with highly conducting and non-conducting walls, less dramatic changes occur. Measurements of velocity, potential and pressure in MHD duct flows confirmed many of the results of the theory for various ducts with constant cross-sectional areas in non-uniform magnetic fields, where $B_0 < 0{\cdot}6T$, $u < 0{\cdot}02$ m/s, $Re < 1{\cdot}7 \times 10^4$, $M < 750$, $N < 33$.

2.3. *Secondary flows*

Chabrerie & Tabeling (Laboratoire de Génie Electrique des Universités Paris vi and xi) presented a theoretical and experimental study of the onset of secondary motions in a flow driven by a magnetic field $\mathbf{B}_0$ and a perpendicular electric current I. The duct is a torus with a rectangular cross-section. $\mathbf{B}_0$ is parallel to the axis of the torus, I is radial and the flow is azimuthal. The asymptotic theory was based on an expansion in power of the small parameter λ (the ratio of the width of the section to the radius of the torus). The first term corresponds to rectilinear flow and the second gives a first approximation to the secondary flow driven by the centrifugal radial pressure gradient. The magnetic field tends to suppress the secondary flow because it has to return through thin Hartmann layers on the walls perpendicular to $\mathbf{B}$. This theory is an improvement on the order of magnitude arguments of Baylis (1971). Experiments were carried out with mercury in a torus (typical width 3 cm) with values of $\mathbf{B}_0$ up to 1·3T and I_0 varying from 10^{-3} to 10^3 A. The electrodes were made of nickel covered with gold; the gold dissolved in the mercury, making a good electrical contact. (It was pointed out that other research groups have not found this necessary, e.g. Baylis & Hunt 1971.) Measurements of voltage drop as a function of I and $\mathbf{B}$ showed that as K/M^2 increased, where K is a Dean number $(= \lambda^{\frac{1}{2}}\, Re)$, first a secondary flow developed and then, at large enough values of K/M^2, Taylor instabilities also became evident.

These results agreed with previous MHD measurements of Baylis (1971). In the limit of large enough current they agreed with the secondary flow measurements in the absence of MHD effects (Ludwieg 1951) and the theory of thermally driven secondary flow of Cheng & Akiyama (1970). In the limit of small enough current at high Hartmann number the results agreed with the theoretical prediction of Baylis & Hunt (1971). In discussion Branover reminded the meeting of the extensive experimental investigations of *pressure-driven* annular flows with *radial* magnetic fields performed at Riga (Branover & Tsinober 1970). Shercliff mentioned that something can be learned about these annular flows if the electric potential is measured along the insulating walls, much as Baylis did.

Alemany & Moreau (Université de Grenoble) presented an analysis of the effect of a rotating magnetic field on the motion of a liquid metal along a circular duct, which extends the earlier work of Moffatt (1965), Dahlberg (1971) and Kapusta (1968). The first class of problems concerns flows in which the pulsations of the electromagnetic forces are negligible compared with the inertial forces; then the essential parameters are the Hartmann number M and the number p of pairs of poles of the applied magnetic field. The most remarkable prediction is that if $p > 2$, a jet must exist in a region near the axis where the magnetic field is weak. This jet is characterized by an equilibrium between the driving pressure gradient and the viscous forces, while in the surrounding core flow the dominant forces are electromagnetic and pressure forces. Of course Hartmann layers are always present near the walls. The second class of problems concerns flows with negligible inertia ($N \gg 1$), so that the pulsation of the electromagnetic forces plays an essential role. In a frame of reference travelling round with the magnetic field these forces are stationary but have an angular periodicity. Then if $M \gg 1$, the flow can be cut up into various flow regions; as the radius increases these are the axial region, the core and the Hartmann layers; as the polar angle varies shear layers in which high velocities can develop are found on the radial lines joining the poles and on their bisectors. Thence it was shown that this pulsation of the electromagnetic forces creates p vortices, the streamlines of which are parallel to the magnetic lines in the core and which close on themselves in the Hartmann boundary layers on the wall of the pipe.

In the discussion, Alemany & Moreau mentioned that the pulsation of the electromagnetic forces produces negative axial velocities, especially if the walls are conducting; in that case most of the flow travels through the shear layers along the lines joining the poles.

Garnier (Université de Grenoble) presented results of a similar investigation, but for the case of magnetic fields travelling *along* a rectangular duct, as in electromagnetic induction pumps. The periodic structure of the electromagnetic forces again created a vortex for each pair of poles (its vorticity now being perpendicular to the direction of the flow). As before the current lines (relative to a moving frame of reference) coincide with the magnetic field lines in the inviscid core flow and have their return paths through the Hartmann boundary layers. If the magnetic field coils facing each other on each side of the duct are such that North faces South, then a jet is induced along the central axis. The asymptotic analysis valid for $N \gg 1$ and $M \gg 1$ is based on dividing the flow into regions.

Cercignani (Politecnico di Milano) described some numerical calculations of an idealization of the MHD motion found in the liquid-metal cathodes used in the electrolytic processing of aluminium salts. The bath of molten aluminium salts is situated below a graphite anode and above the previously purified aluminium, which acts as a cathode. The magnetic fields which drive the motions are induced by the high currents involved. To avoid considering surface waves, a shallow-water approximation was used to analyse motions in a horizontal square bath with a vertical current and a horizontal magnetic field. The computation showed that as the Reynolds number of the flow was increased (by increasing the currents) from 60 to 200 a single vortex flow could develop into 4 vortices.

In the discussion Moreau suggested that since in the molten salt there is a gap between the anode and the containing walls, horizontal, rotational $\mathbf{j} \times \mathbf{B}$ forces can exist which induce vortex motions in the molten salt even though its electrical conductivity is much less than that of molten aluminium. The combined action of this motion with that in the cathode below (discussed by Cercignani) can, on account of the small difference in the densities of the two fluids, lead to instabilities at the interface. Such instabilities are believed to have been the cause of some notorious accidents in the aluminium industry. The literature on these problems is little known: see for example papers by Mead & Ray (1969) and Givry (1967).

2.4. *Waves in MHD duct flows*

Walker (University of Illinois, Urbana) had analysed compression waves in liquid metals in the presence of strong uniform magnetic fields (analogous to the water-hammer problem), a problem that is likely to occur in liquid-metal pumps, generators and fusion-reactor cooling systems. Typically the speed of sound $c \sim 10^3$ m/s, which is much greater than the velocity u induced by a water hammer, so the Mach number $Ma = u/c \ll 1$. Since $R_m \ll 1$, Alfvén waves can be ignored but Ma may be larger than, smaller than, or about equal to R_m. The results presented were for $Ma \gg R_m$. It was also assumed that the Hartmann number $M \gg 1$, so that viscous effects were confined to the walls or to the wave front itself. The jump in velocity across the wave induces circulating electric currents, $\Delta\mathbf{j}$. Assuming that the magnetic field is not too large (i.e. $N \ll Ma^{-1}$), it is found that $\Delta\mathbf{j} = O(\sigma u B_0)$, which produces an additional pressure drop $O(\sigma L B_0^2 u)$. Over a long length of pipe, $O(NMa)^{-1}$ diameters, the wave front is smoothed out. In this distance it may also begin to be curved. No experimental data are yet available to test this theory.

2.5. *Turbulent and unsteady duct flows*

Tananaev (Polytechnic Institute, Leningrad) described pressure and velocity measurements in laminar and turbulent flows along rectangular ducts (14×60 cm^2) with transverse magnetic fields. His latest studies have been on the effect of hemispherical and rectangular bar roughness elements on the duct walls perpendicular to the magnetic field ($\leqslant 1{\cdot}6$ T) (Tananaev 1975). The ducts and the roughness elements were all non-conducting and the height of the elements was $< 0{\cdot}23$ of the half-width. Their effect on the duct flow increases as the magnetic field increases, because in addition to their usual hydrodynamic drag action, they provide a stagnant region of fluid near the wall perpendicular to the magnetic field. This enables the return (or Hartmann) currents to have a lower resistance path. Hence for given flow rate, the electric current in the core increases, giving rise to a greater pressure drop. Tananaev pointed out that their effect is similar to that produced by highly conducting walls perpendicular to the magnetic field. This has an additional effect, as was found by Pitot-tube traverses, of producing large velocity excesses in the boundary layers on the walls parallel to $\mathbf{B}_0$, similar to those predicted by Hunt (1965) for smooth ducts and measured by Gnatyuk & Paramanova (1971). Empirical correlations for the increases in pressure gradient in laminar and turbulent flows were deduced. These came under some criticism because they did not appear to include the effect of the magnetic field on the flow round the roughness elements. If the interaction parameter N, based on the roughness height, is such that $N \gtrsim 2$, the external flow enters the regions between the elements and their strong effect may disappear (Branover & Tsinober 1970).

Lykoudis (Purdue University, Indiana) began by stating that there is a sufficient body of reliable experimental measurements in turbulent MHD duct flows for a semi-empirical theory for shear-flow turbulence in MHD to be critically tested. Using the mixing-length l hypothesis and arguing that l is a function of the MHD damping force $\sigma u B_0^2$ and that the time scale t of an eddy in the thin turbulent boundary layers on the walls perpendicular to the magnetic field is of the order of η/τ_w, where τ_w is the wall shear stress, it follows that $l = l_0 \exp(-A\lambda^2)$, where A is a constant, and $\lambda = M/[Re(\tau_w)^{\frac{1}{2}}/(\frac{1}{2}\rho u^2)^{\frac{1}{2}}]$ (Lykoudis & Brouillette 1967). Lykoudis showed that this hypothesis leads to velocity and Reynolds-stress distributions and pressure drops as functions of M and Re for duct flows which agree with the experiments with transverse magnetic fields undertaken at Purdue (e.g. Brouillette & Lykoudis 1967), and also those with parallel fields undertaken at M.I.T. by Fraim & Heiser (1968) and in Moscow by Kovner & Krasilnikov (1965). The new data on Reynolds stress by Reed at Purdue was interesting and an impressive advance in the detailed measurement of MHD turbulent flows. The mixing-length-damping hypothesis was also applied successfully to heat transfer calculations in pipes with parallel fields (Lykoudis & Andelman 1976). The constant A was the same in all these calculations. Lykoudis had also done some calculations using this method on the motion and the magnetic field in sunspots. Branover commented that he had used Lykoudis's damped-mixing-length hypothesis successfully in a number of investigations (Branover & Tsinober 1970, chap. 4) but had found that it did not work in the case of flow in a circular pipe in a transverse magnetic field.

Rosant (Université de Grenoble) described measurements of mean and fluctuating velocity $\bar{u}$, u' and mean and fluctuating electric field $\bar{e}$, e' for three types of pressure-driven duct flow with transverse magnetic fields. (For the methods of measurement used see Alemany & Rosant, q.v.) Two non-conducting ducts (20×57 mm^2 and 40×22 mm^2) were used with the magnetic field perpendicular to the long side in the first case and parallel in the second. The flow was not fully developed right across the duct; velocity profiles in the plane normal to the magnetic field were found to have maxima near the side walls AA ('M' profiles in the jargon) owing to recirculating currents at the entrance to the duct (Shercliff 1962, p. 72). Rosant found, as one might expect, that these velocity profiles were more unsteady than the core flow. In the core the values of $u'/\bar{u}$ and $e'/\bar{u}B_0$ were similar (owing to the smallness of the electric currents) and both decreased in proportion to each other as the magnetic field increased ($0 < M/Re < 8 \times 10^{-3}$). The reduction was stronger when $\mathbf{B}_0$ was parallel to the long side rather than the short side.

In Rosant's third duct the walls BB perpendicular to $\mathbf{B}_0$ were split into highly conducting halves ($x < 0$) and non-conducting halves ($x > 0$) opposite each other; the walls AA were non-conducting (see figure 1). As expected the velocity for $x < 0$ was very small compared with that for $x > 0$, (for laminar flow the ratio is M^{-1}, where $M < 450$ in this experiment). The experiment was designed to study the instability of the shear layer at $y = 0$; these layers are found to be very unstable with high turbulence. As M increases, the width of the layer decreases and the spectrum becomes more peaky. Rosant suggested that these might correspond to two-dimensional Kelvin–Helmholtz waves.

3. Heat transfer and two-phase flow

Where liquid metals are used in practice (in the metallurgical industry or in reactor cooling systems) more often than not heat is being transferred from or to the fluid. Sometimes the heat transfer may be sufficient for boiling and two-phase flow to occur. The study of the effects of magnetic fields on these processes is only just beginning. *Wilks* (University of Strathclyde) developed an analysis of free convection near a semi-infinite vertical flat plate in a strong uniform transverse magnetic field B_0. Since the Prandtl number $(\eta/\rho)/(\text{thermal diffusivity})$ is very small (10^{-2}) for liquid metals the velocity and temperature fields divide into two regions, an inner viscous layer and an outer thermal layer. The asymptotic analysis reduces to computing the solutions of an ordinary differential equation. As might be expected the computations showed that the heat transfer and the convection velocity decrease as the magnetic field increases. Series solutions to the equations could be used to give adequate approximations. Lykoudis mentioned in discussion that for the case where B_0 varied with the distance x along the plate in proportion to x^{-4}, a similarity solution could be obtained. He had performed experiments which satisfactorily verified the theory (Lykoudis 1962).

Lykoudis (Purdue University) described some problems of boiling and condensation of liquid metals in the presence of magnetic fields; a subject fraught with difficulty as there is still much uncertainty about these phenomena without any magnetic fields. An analysis was presented of the effect of a spherically symmetric ponderomotive force on the growth of a bubble and the heat transfer generated by the bubble in a stationary fluid. This force is a rough representation of a magnetic field. The analysis suggests that the relevant non-dimensional number K is the ratio of the ponderomotive (or electromagnetic) force based on the thermal diffusion time to the typical pressure forces driving the bubble. For example a magnetic field of 10 T decreases the heat transfer by 10% and one of 5 T by 60%. An experiment is underway at Purdue to test these predictions. A theory has also been developed for condensing vapour on a vertical plate in a transverse magnetic field. For laminar flow the classical analysis can be developed to include the effects of a magnetic field, and the results show, as in Wilks's analysis, a reduction in heat transfer as the field is applied.

The question was raised in discussion whether the effects of a magnetic field on boiling would not become greater when the bubbles are sufficiently large for gravity also to be important. It appeared that this aspect of the problem is now being investigated and a new dimensional grouping may be necessary.

Owen, Hunt & Collier (Culham Laboratory) presented two models for predicting the pressure drop in two-phase gas–liquid flows of conducting fluids for large values of the Hartmann number (see Owen, Hunt & Collier 1976). In the first of these models the gas–liquid mixture is treated as a single homogeneous pseudo-fluid with averaged mixture properties; such a 'bubbly' flow is well known in the absence of magnetic fields. It is also observed in the absence of magnetic fields that a flow can develop in which the liquid and gas phases separate into a liquid film on the wall and a gas phase in the core of the pipe. Such a flow was analysed under the assumption of a strong transverse magnetic field so that the film flow is laminar, like a Couette flow with 2 Hartmann layers in each film, while the non-conducting gas in the core is turbulent. The surface shear stress between the two is estimated by conventional friction coefficients. The difference between the two models is that if heat is being supplied the pressure gradient remains constant in the first case but drops in the second case as the film thickness decreases owing to vaporization of the liquid. Experiments have been performed at moderate Hartmann numbers (< 150) and the measured pressure drop is found to lie somewhere between the predictions of these two models.

Kant (Laboratoire de Génie Electrique de Paris (C.N.R.S.)) described a study of the electrical conductivity of two-phase flows under the action of various magnetic fields. This investigation is primarily addressed to the design of induction pumps and generators in two-phase flow. The efficiencies of these devices depend greatly on the conductivity of the fluid, which is very difficult to predict since the velocities of the travelling magnetic field, the liquid, and the bubbles may all be different. Starting with Boltzmann-like equations for distribution functions and adding judicious hypotheses, some speculations for the profiles of conductivity in various situations were obtained. The experimental data presented showed some most interesting effects of magnetic fields. The conductivity in a moving fluid decreases more with a travelling than a stationary magnetic field, and is a function of the 'slip' velocity, the difference between the velocities of the magnetic field and the fluid velocity. The conductivity decreases as the magnetic field increases, probably, Kant believes, owing to interactions between the bubbles. At large magnetic fields the two-phase flow becomes more homogeneous and liquid films become less likely to form on the walls.

4. Instabilities and turbulence

4.1. *Instabilities*

Garnier (Université de Grenoble) showed how the electromagnetic process of confining liquid metals (see §7.2) poses a new stability problem, that of the influence of an *alternating* magnetic field (with frequency $\omega \sim 10^3 - 10^4$ Hz) on the Kelvin–Helmholtz and Rayleigh–Taylor free-surface instabilities. Garnier presented an analysis using the quasi-stationary approximations introduced by Moreau (see §5.2), so that the alternating magnetic field has a similar effect to a mean magnetic field existing in a skin region with thickness $\delta = (\mu\sigma\omega)^{-\frac{1}{2}}$. The fluctuating electromagnetic forces are negligible in this approximation, which requires ω to be large enough for the interaction parameter based on the skin depth to be so small that $N_\delta = (B^2/\rho u_0)(\sigma/\mu\omega)^{\frac{1}{2}} \ll 1$. Results were only presented at those wavenumbers k satisfying the limiting conditions $k\delta \ll 1$ (large wavelength) and $k\delta \gg 1$ (small wavelength). As with steady magnetic fields the influence of the magnetic field was found to be stabilizing or neutral depending on whether the wavenumber was or was not parallel to the velocity.

There was some discussion of the implications of these approximations, which took account of the inductive effects of the alternating field but not its direct dynamical effects. For very low frequencies and high enough magnetic fields where $N_\delta \gtrsim 1$, the fluctuating component of the electromagnetic forces could excite certain instability modes. (Note that N_δ is a measure of the ratio $\Delta u/u_0$, Δu being the variation of velocity over a cycle.)

Plaschko (Tech. Univ. Berlin) also studied the instabilities of the surface of a jet of liquid metal, but in the presence of a parallel uniform and constant magnetic field $\mathbf{B}_0$, restricting himself to an inviscid flow and two-dimensional disturbances. The particular emphasis of the analysis is the spatial evolution along the jet (whose mean profile is also changing) of the amplification of a perturbation as a function of two parameters: the magnetic Reynolds number R_m, and the gradient of the mean velocity $U(y)$, which is characterized by an interaction parameter for the jet S. By transforming the Rayleigh equation to a nonlinear equation which eliminates the gradient of U, the solution of the developing disturbance could more easily be computed. The jet was found to be made more stable by increasing R_m or the jet width or the magnetic field. Some studies of three-dimensional disturbances showed how their growth rates were less inhibited by the magnetic field; another example of the inapplicability of Squire's theorem (1933) to MHD flows (Hunt 1966).

4.2. *Turbulence*

Sulem & Frisch (Observatoire de Nice) considered turbulence initially homogeneous and isotropic, characterized uniquely by the motion of large-scale eddies; they supposed that a strong magnetic field $\mathbf{B}_0$ ($N > 30$) is suddenly applied to the flow at $t = 0$, t being the time. In the first linear phase (Moffatt 1967) the energy decreases as $t^{-\frac{1}{2}}$ and becomes confined to a region of wavenumber space close to the plane perpendicular to $\mathbf{B}_0$, such that $|\cos\theta| \lesssim (\rho/\sigma B_0^2 t)^{\frac{1}{2}}$, θ being the angle between $\mathbf{B}_0$ and the wavenumber S. Thus a quasi-two-dimensional state is attained in which the ratio of length scales parallel to $\mathbf{B}_0$ to those perpendicular to $\mathbf{B}_0$ is of order $N^{\frac{1}{2}}$. But with the decrease of the value of $\cos\theta$ the characteristic time of Joule (or electric current) dissipation increases ($\sim t$), as does the characteristic time for energy transfer between different wavenumbers.

This essentially nonlinear second phase was analysed by adapting to this problem the closure methods and quasi-normality hypotheses used in calculations of the propagation of errors in weather forecasting (Lorenz 1969; Leith 1971; Leith & Kraichnan 1972). The results of the numerical integration appeared to show a reinforcement with time of the quasi-two-dimensional behaviour and a transfer of energy towards large wavenumbers. The Joule dissipation remains of the same order as the energy transfer between wavenumbers. Another interesting result from the closure model was that if the magnetic field is removed, the turbulence becomes three-dimensional again in a time of order $N^{\frac{1}{2}}\ln N$.

Schumann (Kernforschungszentrum, Karlsruhe) presented a direct numerical integration of the Navier–Stokes equations for the decay of homogeneous turbulence in a magnetic field by means of an adaptation of the Fourier transform codes of Orszag & Patterson (1971) (see Schumann 1976). The number of points in the calculation (32^3) limits the validity to moderate Reynolds numbers ($Re < 60$). The initial state is isotropic and evolves freely until a time t_1, when a uniform magnetic field $\mathbf{B}_0$ is applied. It is removed at time t_2. The major characteristics of the turbulence, such as spectra, and global values of the energies parallel and perpendicular to $\mathbf{B}_0$, viscous and Joule dissipation, and skewness coefficients, were calculated. The calculations were performed first taking account of the nonlinear transfer terms and then neglecting them, in order to demonstrate their role in the decay process. At this value of $N(>1)$, these terms had a small tendency to reduce the anisotropy and also led to a small increase in the rate of decay of the total energy.

The result which gave rise to many questions is the rapidity of the decay in total energy; there is a 50% reduction in kinetic energy in a typical decay time l/v when $N = 1$ (l and v being the length and velocity scales of the turbulence). Experimentalists, accustomed to values of Re ten times greater, had never measured decay rates as fast as this.

Branover & *Gershon* (University of the Negev) reviewed a large number of previous experiments on turbulence in MHD duct flows and compared them with new experiments in a rectangular, non-conducting duct especially designed to reduce the 'M' shaped velocity profiles which can disturb the flow on entry; the ratio a/b of the lengths of the walls parallel to the magnetic field AA to those perpendicular BB was about 40 and the duct had a long entrance section. By investigating the difference in the turbulence in the duct due to turbulence generated upstream of the magnetic field and that generated by a grid placed in the duct, they concluded that three-dimensional disturbances are *not* transformed into two-dimensional disturbances when a turbulent duct flow becomes laminar on the application of a magnetic field. The reason why turbulence persists in the core of many MHD duct flows even under strong magnetic fields is because of the instabilities generated by the 'M' shaped entrance velocity profiles (Rosant, q.v.). However, for large enough values of the ratio of the Hartmann number M to the Reynolds numbers Re ($M/Re > 1 \cdot 1 \times 10^{-2}$) any kind of disturbance eventually decays. Shercliff pointed out that in any real experiment even two-dimensional disturbances are damped by a transverse magnetic field, because the eddies must extend from the Hartmann layer on one wall of the duct to the Hartmann layer on the other wall. These layers will suppress vorticity normal to them in a distance down the duct proportional to Re/M.

Moreau (Université de Grenoble) presented some speculations about a general framework for comparing and understanding much of the previous theory on the decay of homogeneous turbulence in a magnetic field. The central question seemed to him to be finding a mechanism for feeding energy to the region in wavenumber space which is undergoing Joule dissipation from the wavenumber region perpendicular to $\mathbf{B}_0$ where there is no Joule dissipation. A simple model was developoped for the turbulence after the magnetic field had been acting for some time: the two-dimensional part (in the plane perpendicular to $\mathbf{B}_0$) is represented by a sum of plane waves and the three-dimensional part is considered as a small perturbation. The model suggests that the wavenumber vectors parallel to $\mathbf{B}_0$ (those with the maximum Joule dissipation) have the most energy supplied to them from the reservoir of two-dimensional motions; the competition between the nonlinear transfer and the Joule dissipation is won by the former. Then from a conjecture about the form of the energy spectrum (in terms of the energy per unit solid angle of wavenumber space), Moreau showed that the spectrum was a *decreasing* function of the angle θ (between $\mathbf{k}$ and $\mathbf{B}_0$) except in the neighbourhood of $\theta = \frac{1}{2}\pi$.

Another speculation concerns the nature of the three-dimensional energy spectrum $E(k)$; by dimensional arguments, by assuming that the ratio of the Ohmic to viscous dissipation ϵ_j/ϵ_ν remains finite as $|\mathbf{B}_0| \to \infty$, and by applying the equilibrium hypotheses of Kolmogorov (1941), it is found that $E(k) \propto k^{-2}$ in the part of the inertial subrange where Ohmic dissipation is significant. This result also follows from the closure hypotheses of Oboukhov (1941). The assumption that ϵ_j/ϵ_ν is finite as $|\mathbf{B}_0| \to \infty$ seems to conform with the reinforcement of the two-dimensional structure and the increase in the Ohmic dissipation decay time found by Moreau & Alemany (1976) and Sulem & Frisch (q.v.).

Alemany (Université de Grenoble) presented an experimetal study of the decay of homogeneous turbulence in a magnetic field. This is realized by moving a grid down through a column of stationary mercury (2.5 m long, 0·2 m in diameter) in a vertical magnetic field B_0 ($<$ 0·28 T) produced by a large solenoid. Thermo-systems hot-film probes are fixed rigidly to the grid at various distances from it. The velocity of the grid can vary from 0·10 to 0·50 m/s, and the mesh sizes G used vary from 25 to 40 mm, so that typical Reynolds numbers based on G are about 600. Thus R_m is much larger than that of the turbulence computed by Schumann (q.v.) and much less than that in the theory of Moreau (q.v.). The measurements from a single hot-film probe were presented which only gave the turbulent velocity component parallel to the applied magnetic field $u_\parallel$. Alemany emphasized that the turbulent velocity field produced by a grid varies with the value of B_0, so there is some difficulty in comparing the effects of varying B_0 on the decay of turbulence. The variance of $u_\parallel$ decays with distance from the grid much less rapidly than predicted by Schumann's computations (by a factor of 5 or 10). Also the correlation lengths parallel to the magnetic field seem to increase with the magnetic field much less than in the linear theory, and in Schumann's numerical integration. Perhaps these discrepancies can be explained by the fact that the theories were not developed for this experimental situation. However the results are in general agreement with a nonlinear theory of self-preserving decay developed by Moreau (1968).

5. Solid objects in MHD flows

5.1. *Fixed obstacles*

Chambarel & *Vivès* (Collège Scientifique Universitaire, Avignon) presented a number of results on pressure distributions around cylinders and spheres (electrically conducting and non-conducting, ferromagnetic and non-ferromagnetic) (Vivès 1974*a*, *b*, 1975). The parameter range explored is $0 < M < 160$; $93 < Re < 11600$. The small gap width of their magnet, 60 mm, imposes such small scales on their experiments that the results cannot easily be related to such theory as exists for the flow around obstacles in ducts in the presence of transverse magnetic fields (for a review see Branover & Tsinober 1970). The cylinder diameter was one half of the duct's width and about 0·6 of its length. The duct walls were non-conducting. They found that for non-conducting cylinders the stagnation pressure increased as $N(=M^2/R)$ increased, but N was not large enough to verify that the pressure becomes proportional to $N^{\frac{2}{3}}$ (Branover *et al.* 1966). For a conducting cylinder the stagnation pressure dropped! They also found that as N increased the pressure minimum moved from around the midpoint to the rear of the cylinder. The total drag obtained by integration of the pressure showed large increases with M, for example, at $M = 160$ there were increases by factors of 28 and 45 for non-conducting and conducting cylinders.

The pressure distributions around spheres were measured on the equatorial circle (perpendicular to $\mathbf{B}_0$) and on various meridional circles. Some of the distributions were rather curious, but no physical interpretation was advanced. The total pressure drag is always larger than without the magnetic field, but the electrical conductivity does not have the same influence as in the case of the cylinder (for $M = 160$). The increases in drag are by factors of 27 and 19 for non-conducting and conducting spheres.

In the discussion it was suggested that the flow over the cylinders would be three-dimensional with much of the flow travelling over the cylinder through thin layers on the side walls, as in a duct with a changing cross-sectional area (Walker, Ludford & Hunt 1972). Consequently Chambarel & Vivès's measurements on the centre-line could be misleading. This three-dimensionality could also explain the unstable nature of the flow, deduced from the fluctuating electric fields. Of this and the other investigations cited (Khalis *et al.* 1966; Tsinober 1970), in only one has there been a successful comparison of theory with experiment (Hunt 1970). The pressure measurements confirmed previous experiments in that separation is suppressed when $N > 1$, and confirmed qualitatively the theory of Moreau (1964) and Buckmaster (1969).

5.2. *Removal of particles*

Three related communications were presented on studies developed at Grenoble on the mean fluid motions in induction furnaces and their use to separate either non-conducting impurities or metal inclusions with a higher conductivity than the melt.

Moreau (Université de Grenoble) began by arguing that such inclusions will have either less or more $\mathbf{j} \times \mathbf{B}$ force acting on them than an equivalent volume of the surrounding liquid, on a principle similar to Archimedes's (an analogy which drew some criticism), but this separating effect is confined to a skin depth (thickness δ) at the sides of the furnace (diameter D) if the frequency of the applied field is high enough (10^4 Hz typically). Steady circulating motions are driven by the rotational $\mathbf{j} \times \mathbf{B}$ force field at the top and at the bottom of the furnace. In these driving regions a jet-like flow is produced with high velocities (2 m/s following Moreau's order of magnitude calculations). On leaving, the flow slowly moves through the core before being returned; the typical velocity in the core should be δ/D times weaker than the jet velocity. Non-conducting particles entrained in the general motion tend to be drawn to the vicinity of the wall where the skin effect concentrates the magnetic field and the $\mathbf{j} \times \mathbf{B}$ forces may be stronger. Moreau's

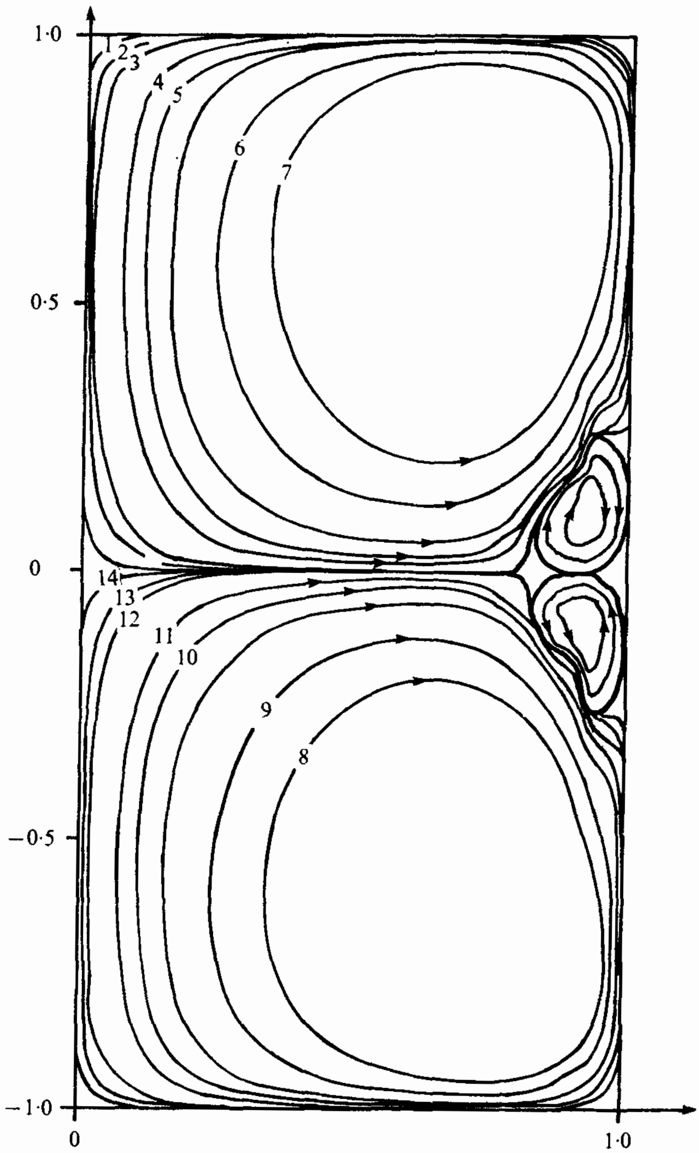

FIGURE 3. Calculated streamlines of the axisymmetric motion induced by the oscillating magnetic field defined in the caption to figure 2. The pattern of these streamlines indicates the pattern of separation seen in figure 2. The y axis is the centre-line of a cylindrical container. The main component of the oscillating magnetic field is vertical.

conclusion was that the future development of the processes based on this mechanism depends crucially on the control of the fluid motion, which has to be able to transport the particles into the region where the separating electromagnetic forces act.

In discussion Block expressed surprise at Moreau's result that the velocity u was proportional to the magnetic field $\mathbf{B}_0$; his experiments had shown that $u \propto B_0^2$, according to an equilibrium between friction and electromagnetic forces.

Moreau claimed experimental support for his result (Tir 1965; Kochetkova *et al.* 1966; Allibert & Driole, q.v.) and argued that the local equilibrium in the jet regions is between inertia and electromagnetic forces, the viscous dissipation being diffused through the whole fluid volume.

Allibert & *Driole* (Université de Grenoble) presented metallurgical experiments on the separation either of non-conducting impurities or metal inclusions. They have taken out some patents on these processes, which should be particularly valuable in processing aluminium and steel (Driole *et al.* 1969, 1975). They also showed an application of this technique to the determination of equilibrium diagrams. An unanswered question arising from these experiments is why separation is obviously visible after solidification when the magnetic field is slowly switched off, but is not when the field is suddenly switched off? Figure 2 (plate 1) gives an example of separation seen after slow solidification.

Khaletzky (Université de Grenoble) presented a numerical study of the flow produced by the forces in an induction furnace. A right circular cylinder was the idealized shape of the furnace. First, the currents and magnetic field induced in the furnace by the high frequency external field (approximately parallel to the axis of the cylinder) were calculated. Then the laminar flow driven by the $\mathbf{j} \times \mathbf{B}$ forces was calculated. These two problems are decoupled because the currents induced by these motions are very small compared with the high frequency induced currents. Depending on the strength and frequency of the driving magnetic field two or four eddies appeared in the furnace (figure 3). With the aid of an assumption about the force on a non-conducting particle in an MHD flow, Khaletzky calculated the likely paths of small non-conducting particles in an induction furnace. It was pointed out in discussion that the assumptions and results of these computations were similar to the analytical calculations of Sneyd (1971) even though he had analysed a uniform fluctuating field perpendicular to the axis of an infinitely long circular cylinder. Sneyd had found 4 recirculating eddies in his analysis.

6. Techniques for local measurements in MHD flows

Berger (Tech. Univ. Berlin) presented calculations of the stagnation pressure on a non-conducting spherical or cylindrical obstacle (e.g. a Pitot tube) placed in a liquid-metal flow at moderately small or moderately large values of R_m, the magnetic Reynolds number. General expansions, but not specific results, were developed in powers of R_m and $N(<1)$, and R_m^{-1} and N for the two ranges of R_m. The practical objective was to infer velocities from Pitot tube measurements in a proposed liquid-metal generator where $0{\cdot}06 < R_m < 6$. Berger mentioned the well-known result that for a cylindrical tube perpendicular to the flow and parallel to the magnetic field, the field does not affect the flow (Shercliff 1965, p. 86) and therefore no correction is required if the tube is used as a Pitot tube. Branover commented that for the case where $N \gg 1$ his experiments had confirmed dimensional arguments that the Pitot pressure was proportional to $N^{\frac{2}{3}}$, assuming $R_m \ll 1$ (Branover *et al.* 1966). An additional effect not considered by Berger was the hydrostatic pressure gradient created in the interior of the Pitot tube between the orifice and the manometer by electric currents passing through the tube in the presence of a transverse magnetic field.

Rosant & Alemany (Université de Grenoble) presented some comparisons between different methods of measuring the flow. (Pitot tube, hot-film anemometer, potential-difference probe.) In the case of a Pitot tube, a double correction was used, first for the electromagnetic pressure at the stagnation point, and second for the influence of the electric field on the interior of the tube, which is greatest when the tube is non-conducting. A variant of the Pitot tube was presented which was a cylinder parallel to $\mathbf{B}_0$; if the flow is truly two-dimensional around the cylinder then no correction is necessary (Berger, q.v.). It was shown that if the hot-film probes were covered only with a thin layer of quartz, then they were sensitive to variations in pressure if the mercury was not very clean; the use of a deposit of gold a few microns thick reduces this effect, but the deposit is quickly eroded. (This precaution is not considered necessary by Lykoudis's group or many other experimental groups using hot films.) Rosant's conclusion was that the simplest and most reliable measurement of *mean* velocities in an MHD flow is that of the electric field. (This method is not practical everywhere because in some regions, such as Hartmann layers, the electric field is constant while the mean velocity changes.) But for measuring velocity fluctuations, these probes give only qualitative information because the relation between velocity and electric field fluctuations is rather uncertain. Therefore hot-film probes remain the only quantitative means of measuring fluctuating velocities despite their many faults. Numerous references were given on the hot-film probe: Sabjen (1965), Malcolm (1969), Gardner & Lykoudis (1971), and Robinson & Larrson (1973); and on the potential-difference probe: Branover *et al.* (1970), Kit (1970); on the Pitot tube: Gnatyuk & Paramonova (1969) and Branover *et al.* (1966). An example of a flow where a detailed comparison was made between a theoretically predicted profile and measurements by all three types of probe was described by Hunt & Malcolm (1968) and Hunt & Stewartson (1969).

Chambarel, Ricou and Vivès (Collège Scientifique Universitaire, Avignon) have developed a quite new technique for measuring velocity based on two successive measurements, first of $\mathbf{j}/\sigma$, $\mathbf{j}$ being the current density, and second $\mathbf{E}$, the electric field. Two potential differences are measured between platinum rings fixed at each end of a small stainless steel circular tube (axial length 4·5 mm, diameter 1·5 mm and wall thickness 0·1 mm). The first measurement is made in a stationary fluid with the same electric field as in the moving-fluid experiment; this gives a local value of $\mathbf{j}/\sigma$. Then the potential difference between the ends of the probe is made in the moving fluid, giving a local value of $\mathbf{E}$; by taking the difference a signal proportional to the velocity is obtained. The technique has been proved by calibrating the device in a Couette flow, measured previously with a Pitot tube. It will be applied in complex flows. They have methods of measuring velocities of the order of 10^{-2} mm/s, pressures of the order of 10^{-5} mm mercury, and, in the presence of a magnetic field of 0·12 T, they measure potential differences of order of 10^{-9} V. The device has been used to measure the laminar velocity profiles in MHD circular Couette flow, and shows satisfactory agreement with the theory.

7. Industrial applications

7.1. *Electromagnetic flow meters*

As Shercliff remarked while chairing this session, electromagnetic flow meters are the oldest application of MHD. Faraday tried to measure the flow of the Thames a year or two after he had first propounded his laws of electromagnetic induction.

Bevir (U.K. Atomic Energy Authority, Culham) discussed some theoretical aspects of finding volume flow rates Q of liquid metals in pipes by imposing a magnetic field $\mathbf{B}_0$ and then measuring the magnetic field $\mathbf{B}_i$ induced by the electric currents generated by the motion $\mathbf{u}(\mathbf{x})$ of the liquid through the magnetic fields. The theoretical question is whether it is possible to construct a flow meter and magnetic field coils such that $\mathbf{B}_i$ at the measuring point $\mathbf{x}_m$ depends upon Q, but not on the form of the velocity profiles $\mathbf{u}(\mathbf{x})$. In the linear regime (when $R_m \ll 1$ and $|\mathbf{B}_i| \ll |\mathbf{B}_0|$), $\mathbf{B}_i$ is proportional to $\mathbf{B}_0$ and to μ, L and σ. Bevir explained how the theory depends on the idea of a weight (or Green's)function $\mathbf{W}(\mathbf{x}_m, \mathbf{x})$ which was originally developed for induced-voltage flow meters by Shercliff (1962, p. 27) for two dimensions and by Bevir (1970) for three dimensions.† Then the signal is expressed as a volume integral over the pipe:

$$\text{signal at} \quad \mathbf{x}_m = \int_{\text{volume}} \mathbf{W}(\mathbf{x}_m, \mathbf{x}) \,.\, \mathbf{u}(\mathbf{x})\, d\mathbf{x}.$$

If the normal component of $\mathbf{u}$ on the surface of the duct is zero and $\operatorname{div} \mathbf{u} = 0$, then $\mathbf{W}$ must satisfy the conditions that, for given $\mathbf{x}_m$, $\operatorname{curl} \mathbf{W} = 0$ and $|\mathbf{W}| \to 0$ as $|\mathbf{x} - \mathbf{x}_m| \to \infty$. In an induced-voltage flow meter a reciprocal theorem relates $\mathbf{W}$ to $\mathbf{B}_0$ and to the current in the flow due to unit current passed through the electrodes. Whereas in an induced-field flow meter $\mathbf{W}$ is related to $\mathbf{B}_0$ and to the effects of unit current passed through the pick-up coils on (*a*) the induced field in the flow (at $\mathbf{x}_m$) and (*b*) on the induced field in the pick-up coils. Using these general ideas it can be shown that in an axisymmetric flow meter $\mathbf{B}_i(\mathbf{x}_m)$ is proportional to Q only if $\mathbf{B}_0$ is produced by coils *within* the flow, so that the flow must pass through an annulus around the magnetic coils. However devices which do not satisfy this criterion can be designed to be reasonably insensitive to the velocity profile, such as the system proposed by Baker (1970). One question raised was whether $\mathbf{B}_0$ in the fluid could be adapted to its optimum form by using the skin effect. This possibility does not seem to have been studied.

Robinson (A.B. Atomic Energi, Nykoping) described some of the experiments of the MHD liquid-metal research group at Nykoping. Their previous work had been on rotating flows (Robinson 1973), but now they are studying various problems of flow metering and flow control for the non-ferrous metals industry. Induced voltage flow meters are unsatisfactory because the hot liquid metal produces a deposition of insulating oxide on the walls. Consequently an induced-magnetic-field flow meter is the most appropriate. An experimental mercury rig was described for testing such a flow meter. The intensity of the induced magnetic

† See also Gammerman & Mezhburd (1971).

field increased as the frequency of the applied magnetic field increased up to a critical value (2000 Hz) when it decreased. This effect must have been due to field exclusion because at this frequency the 'skin depth' was about equal to the pipe diameter. Some unusual methods were described for controlling the flow of a conducting liquid out of a large cylinder through a narrow channel by applying a strong non-uniform transverse magnetic field and by varying the shape of the duct. For some outlet ducts Robinson was surprised to find no effect and in others a strong effect on the flow rate when the magnetic field was applied. In the discussion Branover remarked that many of the effects Robinson found could have been predicted from previous experimental work on flows in various duct shapes and entry flows, particularly from that done by the group at Riga. Since in these flows $N \gg 1$, the theory outlined by Holroyd & Hunt (q.v.) would also have been applicable, particularly that relating to non-uniform magnetic fields.

Block (Inst. Eisenhüttenkunde, Aachen) reviewed the applicability of 5 electromagnetic methods of measuring the flow of molten metal (see also the monograph by Kisis 1968). The first two methods involve measuring the induced voltage produced by the motion of the liquid through a constant or alternating magnetic field. Usually the electrodes are solid, as for example in *e/m* flow meters used for measuring the flow rates of water or even sewage. But in molten metal it is better if the solid electrodes are withdrawn a little way down their containing tubes each side of and perpendicular to the measuring duct. Then a small standing eddy exists in each tube between the solid electrode and the flow, an ingenious idea which seems to work well, after being first tested in a mercury flow circuit. The other 3 methods rely on eddy currents.† The flow rate is proportional to the phase difference between the induced and the imposed magnetic field, if the latter is a travelling field. Or the deformation of the imposed magnetic field can be measured (Bevir, q.v.). Or a fluctuating magnetic field can be imposed at one location in the flow $B_1(t)$ and then at a point downstream the magnetic field can be measured $B_2(t)$. By measuring the cross-correlation with time delay, i.e. $\overline{B_1(t)\, B_2(t=\tau)}$, between the signals the flow rate can be inferred. These three methods were tested (i) in a molten-steel flow circuit, the steel being melted in an induction furnace and driven round by a powerful rising stream of air bubbles, (ii) at a rotating aluminium disk and (iii) in a stream (0·5–1 m/s) of molten steel with a free surface in a channel of sand. Block's conclusion was that the phase difference and correlation methods are the best for measuring the flow of molten steel. A question was raised whether these methods are sensitive to temperature effects. Apparently the effect of temperature on conductivity in induced-field magnetic flow meters can be avoided or reduced by choosing the appropriate frequency, at which the increase in the signal with frequency is balanced by the reduction in the signal as the imposed field is excluded.

† Most of the delegates who were not French did not know that in France the discovery of these currents is attributed to Foucault and that these currents bear his name.

7.2. *Application of electromagnetic flows*

Shercliff (University of Warwick, Coventry) began by pointing out that one of MHD's most important and characteristic phenomena, the Alfvén wave, had never been applied in technology (Shercliff 1976). Alfvén waves might be used as a mechanism for extracting energy from a fluid 'fly-wheel'. The disadvantage of a solid fly-wheel as an energy storage device is that it acts like a capacitor in that its voltage falls as energy is extracted. However, a fluid flowing in an annulus with an axial magnetic field between conducting electrodes (walls AA) and non-conducting walls (BB) would not have this disadvantage. Electrical energy supplied slowly could be extracted rapidly by connecting the electrodes across a suitable load. As the fluid slowed down, an Alfvén wave would travel upwards from one of the walls BB. At each point the fluid would halve its speed instantaneously as the wave passed. When the wave had returned to its original wall BB, all the energy would have been extracted. To avoid the viscous and electrical losses associated with a small device, any practical device would have to be huge; for example a 4 m $\times$ 4 m annular channel of sodium. Other parameters would be $B_0 \sim 1$T, $u \sim 30$ m/s, total kinetic energy $0{\cdot}36 \times 10^9$ J, and the power extracted would be $1{\cdot}3 \times 10^9$ W over 0·27 s. The best laboratory investigation of Alfvén waves in liquid metals is that of Jameson, who studied progressive and standing waves in an annulus of sodium. Only his work on standing waves was published (Jameson 1964); the measurements on progressive waves, which confirmed the theory very well, remain in his unpublished Ph.D. thesis.

There would be many advantages in the processing of hot liquefied metals if the diameter of a metal stream could be reduced without using solid walls on which the metal might solidify (for example, regulation of flow rate, reduction of damage to walls, better wire manufacture, etc.). *Garnier* (Grenoble) explained the patented technique being studied at Grenoble (Moreau & Garnier 1975). A vertical cylindrical stream falls through a solenoid that produces a high frequency magnetic field B_z along the stream. Because of azimuthal eddy currents j_θ (or the skin effect) the field is confined to a thin layer on the outside of the stream. The strong radial $\mathbf{j} \times \mathbf{B}$ force means the pressure in the centre is higher than atmospheric. At some downstream position a copper annulus placed around the stream draws off the eddy currents into itself from the liquid metal stream. This means the $\mathbf{j} \times \mathbf{B}$ force drops to zero in the stream; its pressure *drops*; so by Bernoulli and continuity the diameter of the stream decreases. (There is some similarity to a Borda mouthpiece.) This device is different from that used in the U.S.S.R. where $\mathbf{j} \times \mathbf{B}$ forces directly cause the diameter to be smaller (Getselev 1971; see also Moreau 1974). The process here is more indirect. Experiments with mercury are under way to verify the idea.

The paper of Hühns & Djamali-Schami (Tech. Univ. Berlin), presented by *Berger*, treated the purely electrodynamic problem of calculating the changes in the magnetic field and the $\mathbf{j} \times \mathbf{B}$ forces produced by motion of a solid free jet travelling along the centre of an MHD induction generator. The velocity was assumed uniform, and the effects of finite length were ignored, but detailed attention was paid to effects of finite R_m and the difference between axisymmetric dipole and quadrupole travelling fields. From solutions involving Fourier series and Bessel functions, the mean and fluctuating $\mathbf{j} \times \mathbf{B}$ forces were calculated. If the jet was liquid it would be necessary to know if it would remain central under these forces. It was inferred that dipole fields would be better in this respect.

Davidson & Thatcher (U.K. Atomic Energy Authority, Warrington) first presented a brief review of electromagnetic devices which are or may be used in the sodium cooling circuits of nuclear-fission reactors. Electromagnetic pumps are of two main types, conduction and induction; in the first, conduction type, potential differences drive electric currents (a.c. or d.c.) between conducting walls, which act with a magnetic field to pump the fluid. In the second, electric currents and $\mathbf{j} \times \mathbf{B}$ forces are induced by high-frequency magnetic fields produced by coils surrounding the duct. The efficiency of *e/m* pumps ($< 40\%$) is much less than that of centrifugal pumps (70–80%), which is the reason for the latter being used in the main cooling circuits. The greater reliability of *e/m* pumps means that they are currently used for ancillary flows where the flow rate is low ($10^{-3}\,\mathrm{m^3/s}$). If reliability comes to be regarded as the more critical criterion then the *e/m* pump might be preferred.

With an *electromagnetic flow coupler* one flow circuit may be used to drive a second circuit without moving parts or any contact between the fluids (which may be highly radioactive). The device also helps to avoid thermal shock if one circuit suddenly cools or overheats. It consists of two rectangular ducts, one from each circuit placed in a magnetic field. The first duct acts as a generator and produces a current which circulates through the second duct, which then acts as a pump. An efficiency of 50 to 70% is possible in theory. *Electromagnetic brakes* are a possible way of quickly slowing down the coolant, whose speed may be as high as 50 m/s, without sudden pressure waves damaging the system. It was suggested in discussion that self-exciting brakes might be considered, as these can prevent flow in one particular direction, unlike a brake with an external magnetic field. In sodium, induced-field flow meters are the most appropriate because of the high value of R_m (< 20) owing to the large diameter of the pipes and the high speeds. Thatcher presented experimental results at $R_m = 5$ where the induced magnetic field was comparable with the imposed field and the field was swept downstream. Although the experiments were in circular pipes, the results agreed well with theory for a two-dimensional flow (Thatcher 1971). The unsolved but particularly important problem of monitoring flows in the myriad of pipes of prototype reactors was also mentioned.

7.3. *Power generation*

Radebold (Tech. Univ. Berlin) gave a graphic account of the technical problems being overcome in constructing and running their liquid-metal MHD generator. The generator is powered by hot potassium vapour at 900 °C which condenses and produces a liquid-metal flow through a travelling magnetic field generator, designed to produce 10–15 kW at a value of R_m of 4·0 to 6·0. It is modelled on the original design of Jackson and Peirson. However, so far the output has been limited by severe noise which is not understood; it may be connected with turbulence in the flow. Future plans involve changing the working fluid to a ferromagnetic fluid, which it is thought will produce a smoother output, and to pass the conducting or ferromagnetic fluid through the generator in the form of a free jet (Hühns & Djamali-Schami, q.v.).

8. Conclusions

Some general conclusions about the most useful direction of future MHD research and about the applications of MHD seemed to emerge from the conference.

(*a*) A good deal of previously published research on MHD may be of use to research workers involved in applying MHD to industrial problems. For example, the large number of MHD duct flow studies with different shapes of duct and different wall conductivities, in laminar and in turbulent flow regimes. The recent review published by ERDA (1975) and Lielausis (1975) may be of some help in making previous work available.

(*b*) Many of the experiments now being performed are generating new theoretical problems that need study. This would be more worthwhile than dotting the i's and crossing the t's of the previous generation of MHD problems, such as laminar duct flows with constant magnetic fields. Some examples of these new problems are:

(i) duct flows in non-uniform magnetic fields where the interaction parameter $N > 1$, and in some cases $R_m > 1$,

(ii) effects of travelling or fluctuating magnetic fields on the flow,

(iii) forces on solid particles in an MHD flow, where the particles may or may not have the same conductivity as the fluid,

(iv) secondary motions generated in MHD flows by inertial effects (such as centrifugal forces) or by oscillating magnetic fields,

(v) free-surface effects in MHD flows which are particularly important in metallurgical processing in the presence of magnetic fields.

(*c*) There seems to be some scope for the ingenious application of the established concepts and findings of MHD to advancing the technology of the metallurgical industry. The conversion of the industry to continuous casting makes MHD methods more attractive as a means of controlling, mixing and purifying metals. Some examples were given at this Colloquium.

(*d*) In association with developing the applications of MHD, it is necessary to continue with small-scale experiments in the laboratory to test out new ideas for applying MHD, and to develop new ones. Mercury still appears to be the most convenient working fluid for this purpose, unless a high value of R_m is necessary.

Even now there are remarkably few steady laminar shear flows or boundary layers for which theoretical and experimental velocity and electric potential profiles have been compared and found to agree to within a few per cent. A few more such experiments are highly desirable, to substantiate MHD theories and to verify measurement techniques. Unsteady flows and transition to turbulence have been even less precisely measured and calculated.

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REFERENCES

(† Denotes a paper given at the colloquium)

ALBERNY, R., BADKER, L., BIRAT, J-P, GOSSELIN, P. & WANIN, M. 1974 Quality improvement of strand-cast billets through electromagnetic stirring. *Electric Furnace Proc., Cincinnati Metall. Soc. A.I.M.E.* **31**, 237.

ALEMANY, A. † Experimental study of homogeneous turbulence in MHD.

ALEMANY, A. & MOREAU, R. † Flow of an electrically conducting liquid in the presence of a rotating multipolar magnetic field.

ALEMANY, A. & ROSANT, J. M. † Observations on the use of hot-film anemometers in mercury.

ALLIBERT, C. & DRIOLE, J. † Some metallurgical examples of the action of a magnetic field on the separation of metallic phases.

BAKER, R. C. 1970 Linearity of motion induced magnetic field flowmeter. *Proc. I.E.E.* **117**, (3), 629.

BAYLIS, J. A. 1971 Experiments on laminar flow in curved channels of square section. *J. Fluid Mech.* **48**, 417.

BAYLIS, J. A. & HUNT, J. C. R. 1971 MHD flow in an annular channel; theory and experiment. *J. Fluid Mech.* **48**, 423.

BERGER, E. † The influence of the magnetic pressure on Pitot-tube readings in liquid-metal MHD flow.

BEVIR, M. K. † Ideal induced field flow meters.

BEVIR, M. K. 1970 The theory of induced voltage electromagnetic flowmeters. *J. Fluid Mech.* **43**, 577.

BLOCK, F. R. † Measuring the velocity of electrically conducting solids and liquids.

BRANOVER, G. G., GEL'FYGAT, YU. M., KIT, L. G. & PLATNIEKS, I. A. 1970 Effect of a transverse magnetic field on the intensity profiles of turbulent velocity fluctuations in a channel of rectangular cross section. *Magn. Gidro.* **6**, 336 (MHD **6** (3), 41).

BRANOVER, G. G., GEL'FGAT, YU. M., TSINOBER, A. B., SCHTERN, A. B. & SHCHERBININ, E. V. 1966 The application of Pitot and Prandtl tubes in MHD experiments. *Magn. Gidro.* **2**, 1, 98 (MHD **2**, 55).

BRANOVER, G. G. & GERSHON, P. † Experimental verification of the hypothesis of two-dimensional turbulence in MHD flows.

BRANOVER, G. G. & TSINOBER, A. 1970 *Magnetohydrodynamics of Incompressible Media.* Moscow: Nauka.

BROUILLETTE, E. C. & LYKOUDIS, P. S. 1967 Magneto-fluid-mechanic channel flow. I. Experiment. *Phys. Fluids*, **10**, 955. II. Theory. *Phys. Fluids*, **10**, 1002.

BUCKMASTER, J. 1969 Separation and magnetohydrodynamics. *J. Fluid Mech.* **38**, 481.

CERCIGNANI, C. † On the dynamics of a liquid layer subjected to magnetic forces.

CHABRERIE, J-P., FOURNET, G. & MAILLEFERT, A. 1976 Studies of plane parallel turbulent flows with moving walls. *Rev. Phys. Appl.* **11**, 353.

CHABRERIE, J-P. & TABELING, P. † Study of laminar to turbulent transition in an MHD annular flow.

CHABRERIE, J-P., & TABELING, P. † Rectilinear MHD flow in rectangular cross-sections with sliding walls.

CHAMBAREL, A., RICOU, R. & VIVÈS, CH. † Electrical method for determining values of local electromagnetic parameters and the velocity in an MHD flow.

CHAMBAREL, A. & VIVÈS, CH. † Pressure distributions on cylindrical obstacles with various electrical and magnetic properties in the presence of a magnetic field.

CHENG, K. C. & AKIYAMA, M. 1970 Laminar force convection heat transfer in curved rectangular channels. *Int. J. Heat Mass Transfer*, **13**, 471.

COOK, L. P., LUDFORD, G. S. S. & WALKER, J. S. 1972 Corner regions in the asymptotic solution of $\epsilon\nabla^2 u = \partial u/\partial y$ with reference to MHD duct flow. *Proc. Camb. Phil. Soc.* **72**, 117.

DAHLBERG, E. 1971 On the action of a rotating magnetic field on a conducting liquid. *Atomic Energi, Sweden, Rep.* AE 447.

DAVIDSON, D. F. & THATCHER, G. † Sodium electrotechnology at UKAEA Reactor group.

DRIOLE, J., ALLIBERT, C., BONNIER, E. & WICKER, A. 1969 Procédé de séparation d'un corps en phase solide hors d'une matrice en phase liquide. *French Patent*, no. 69.164.23.

DRIOLE, J., ALLIBERT, C., BONNIER, E. & WICKER, A. 1975 *2nd Suppl. Cert. to patent request*, no. 69.164.23.

ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION 1975 *Magnetohydrodynamic Power Generation and Theory – a Bibliography*. Available from Nat. Tech. Information Service. TID-33-56.

FRAIM, F. W. & HEISER, W. H. 1968 The effect of a strong longitudinal magnetic field on the flow of mercury in a circular tube. *J. Fluid Mech.* **33**, 397.

GAMMERMAN, M. YA. & MEZHBURD, V. I. 1971 Weight functions for electromagnetic flow meters in the 3-dimensional approximation. *Magn. Gidro.* **7** (3), 130 (MHD **7**, 406).

GARDNER, R. & LYKOUDIS, P. 1971 Magneto-fluid-mechanic pipe flow in a transverse magnetic field. Part 1. Isothermal flow. *J. Fluid Mech.* **47**, 737.

GARNIER, M. † Kelvin–Helmholtz and Rayleigh–Taylor instabilities in the presence of alternating magnetic fields.

GARNIER, M. † Duct flows in the presence of a travelling magnetic field.

GARNIER, M. † Electromagnetic confinement of liquid metals.

GETSELEV, Z. N. 1971 Knibyskevski Metull. Zarod Inreni V.I. Lenina. *French Patent*, no. 71, 41163.

GIVRY, J. P. 1967 Computer calculation of magnetic effects in the batch of aluminium cells. *Trans. Met. Soc. A.I.M.E.* **239**, 1161.

GNATYUK, V. V. & PARAMONOVA, T. P. 1969 Calibration of Pitot tubes in a transverse magnetic field. *Magn. Gidro.* **5** (4), 143 (MHD **5**, 96).

GNATYUK, V. V. & PARAMONOVA, T. A. 1971 Effect of wall conductance on the velocity profile in a pipe. *Magn. Gidro.* **7** (1), 145 (MHD **7**, 126).

HERVÉ, R. & POIRIER, J. † Study of rectilinear fluid flow in a rectangular duct one of whose walls is highly conducting, the others being non-conducting.

HOLROYD, R. J. 1976 Magnetohydrodynamic duct flows in non-uniform magnetic fields. Ph.D. dissertation, University of Cambridge.

HOLROYD, R. J. & HUNT, J. C. R. † A review of MHD flows in ducts with changing cross-sectional areas and non-uniform magnetic fields.

HÜHNS, T. & DJAMALI-SCHAMI, D. † Comparison of three types of free-jet MHD induction generators; three-dimensional field distributions and performance.

HUNT, J. C. R. 1965 Magnetohydrodynamic flow in a rectangular duct. *J. Fluid Mech.* **21**, 577.

HUNT, J. C. R. 1966 On the stability of parallel flows with parallel magnetic fields. *Proc. Roy. Soc.* A **293**, 342.

HUNT, J. C. R. 1970 Bluff body drag in strong transverse magnetic field. *Magn. Gidro.* **6** (1), 35 (MHD **6**, 30).

HUNT, J. C. R. & HANCOX, R. 1971 The use of liquid lithium as a coolant in a toroidal fusion reactor. Part 1. Calculation of pumping power. *U.K. Atom. En. Auth. Rep.* CLM-R115.

HUNT, J. C. R. & LUDFORD, G. S. S. 1968 Three-dimensional MHD duct flows with strong transverse magnetic fields. Part 1. Obstacles in constant area channel. *J. Fluid Mech.* **33**, 693.

HUNT, J. C. R. & MALCOLM, D. G. 1968 Some electrically driven flows in MHD. Part 2. Theory and experiment. *J. Fluid Mech.* **33**, 775.

HUNT, J. C. R. & SHERCLIFF, J. A. 1971 Magnetohydrodynamics at high Hartmann number. *Ann. Rev. Fluid Mech.* **3**, 37.

HUNT, J. C. R. & STEWARTSON, K. 1965 Magnetohydrodynamic flow in rectangular ducts. II. *J. Fluid Mech.* **23**, 563.

HUNT, J. C. R. & STEWARTSON, K. 1969 Some electrically driven flows in MHD. Part 3. The asymptotic theory for flow between circular electrodes. *J. Fluid Mech.* **35**, 225.

JAMESON, A. 1964 Magnetohydrodynamic waves. Ph.D. dissertation, University of Cambridge.

KANT, M. † Contribution to the study of the electrical continuity of two-phase flows under the action of a magnetic field.

KAPUSTA, A. B. 1968 Motion of a conducting fluid under the action of a rotating magnetic field. *Magn. Gidro.* **4**, 71 (MHD **4**, 71).

KHALETZKY, D. † Numerical study of the motions in an induction furnace used for separating impurities.

KHALIS, K. E., SLYUSAREV, N. M., TSINOBER, A. B. & SCHTERN, A. G. 1966 Resistance of bluff bodies at high Stewart numbers. *Magn. Gidro.* **2**, 152 (MHD **2**, 93).

KISIS, A. YA. 1968 Electromagnetic measurement of the parameters of MHD processes. *Latvian Acad. Sci.*

KIT, L. G. 1970 Turbulent velocity fluctuation measurements using a conduction anemometer with a three electrode probe. *Magn. Gidro.* **6**, 41 (MHD **6**, 480).

KOCHETKOVA, G. YA., STOLOV, M. YA., TIT, L. L. & CHAIKIN, P. M. 1966 The circulation of metal in an induction furnace. *Magn. Gidro.* **2** (2), 139 (MHD **2**, 85).

KOLMOGOROV, A. N. 1941 The local structure of turbulence in an incompressible viscous fluid for very large Reynolds numbers. *Dokl. Akad. Nauk USSR*, **30**, 301.

KOVNER, D. S. & KRASIL'NIKOV, E. IN. 1965 An experimental investigation of turbulent flow of electroconductive liquid in a pipe in a parallel magnetic field. *Dokl. Akad. Nauk USSR*, **163**, 1096.

KULIKOVSKII, A. G. 1968 Slow steady flows of a conducting liquid at large Hartmann numbers. *Izv. Akad. Nauk USSR, Mekh. Zhid. i Gaza*, **3** (2), 3 (*Fluid Dynamics*, **3**, 1).

KULIKOVSKII, A. G. 1973 Flows of a conducting incompressible liquid in an arbitrary region with a strong magnetic field. *Mekh. Zhid. i Gaza*, **8** (3), 144 (*Fluid Dynamics*, **8**, 462).

LEITH, C. E. 1971 Atmospheric predictability and two-dimensional turbulence. *J. Atmos. Sci.* **28**, 145.

LEITH, C. E. & KRAICHNAN, 1972 Predictability of turbulent flows. *J. Atmos. Sci.* **29**, 1041.

LIELAUSIS, O. A. 1975 Liquid metal magnetohydrodynamics. *Atomic Energy Review*, **13**, 527.

LORENZ, E. N. 1969 The predictability of a flow which possesses many scales of motion. *Tellus*, **21**, 289.

LUDWIEG, 1951 Die ausgebildete Kanalströmung in einem rotierenden System. *Ing. Arch.* **19**, 563.

LYKOUDIS, P. † Magneto-fluid-mechanics liquid-metal turbulent shear flows.

LYKOUDIS, P. † Some elementary magneto-fluid-mechanic problems for two-phase fluids.

LYKOUDIS, P. 1962 Natural convection of an electrically conducting fluid in the presence of a magnetic field. *Int. J. Heat Mass Transfer*, **5**, 23.

LYKOUDIS, P. & ANDELMAN, M. 1976 Liquid-metal heat transfer in pipes with aligned magnetic fields. *Trans. Am. Nuc. Soc. Ann. Meeting.* **21**, 36, June 1975.

LYKOUDIS, P. & BROUILLETTE, E. C. 1967 Magneto-fluid-mechanic channel flow. II. Theory. *Phys. Fluids*, **10**, 1002.

MALCOLM, D. G. 1969 Some aspects of turbulence measurement in liquid mercury using cylindrical quartz insulated hot film sensors. *J. Fluid Mech.* **37**, 701.

MEAD, J. J. & RAY, R. Y. 1969 Electromagnetic problems in aluminium reduction cells, *TMS-AIME Annual Meeting, Washington D.C.*

MOFFATT, H. K. 1964 Electrically driven steady flows in MHD. *Proc.* 11th *Int. Cong. Appl. Mech.* p. 946. Munich.

MOFFATT, H. K. 1965 On fluid flow induced by a rotating magnetic field. *J. Fluid Mech.* **22**, 521.

MOFFATT, H. K. 1967 On the suppression of turbulence by a uniform magnetic field. *J. Fluid Mech.* **28**, 571.

MOFFATT, H. K. 1973 MHD phenomena in rotating fluids. Report of the NATO advanced study institute. *J. Fluid Mech.* **57**, 625.

MOREAU, R. † Reflections on the possibilities of electromagnetic separation.

MOREAU, R. † Some general ideas on homogeneous turbulence in the presence of a uniform magnetic field.

MOREAU, R. 1964 The effect of a transverse magnetic field on separation. *C. R. Acad. Sci.* **258**, 1732.

MOREAU, R. 1968 On magnetohydrodynamic turbulence. *Proc. of Symp. on Turbulence of Fluids and Plasmas. Polytech. Inst. of Brooklyn*, p. 359.

MOREAU, R. 1974 Procédé electromagnetique de controle du digazage des métaux liquides par jet sous vide. *French Patent*, no. 74, 21569.

MOREAU, R. & ALEMANY, A. 1976 Aspects spécifiques de la turbulence homogène MHD aux faibles nombres de Reynolds magnetiques. *J. Physique. Suppl.* **1, 37**, pp. C1–101.

MOREAU, R. & GARNIER, M. 1975 Dispositifs électromagnetique de confinement des métaux liquides. *French Patent*, no. 75, 21075.

OBOUKHOV, A. M. 1941 On the energy distribution in the spectrum of a turbulent flow. *Dokl. Akad. Nauk USSR*, **32**, 19.

ORSZAG, S. A. & PATTERSON, G. S. 1971 *Numerical Simulation of Turbulence. Statistical Models and Turbulence.* Springer.

OTTE, F. † Numerical solution of a two-dimensional liquid-metal MHD duct flow for finite magnetic Reynolds numbers in the presence of a strong transverse magnetic field varying in the flow direction.

OWEN, R. G., HUNT, J. C. R. & COLLIER, J. G.† MHD pressure drop in ducted two-phase flows. (To appear in *J. Multiphase Flow* 1976.)

PLASCHKO, P. † The influence of continuously varying velocity profiles on the instability of spatially growing disturbances in a liquid metal MHD jet in the presence of a strong longitudinal magnetic field.

RADEBOLD, R. D. † Preliminary experimental results from the MHD staustrahlrohr (convertor).

REED, C. 1976 Ph.D. dissertation, Purdue University.

ROBINSON, T. † Experiments on flow measurement in a mercury loop.

ROBINSON, T. 1973 An experimental investigation of a magnetically driven rotating liquid-metal flow. *J. Fluid Mech.* **60**, 641.

ROSANT, J. M. † Turbulent flows in rectangular ducts at large Hartmann numbers.

ROSANT, J. M. † Comparison of different methods of local velocity measurement.

SABJEN, M. 1965 Hot wire anemometers in liquid mercury. *Rev. Sci. Instr.* **36**, 945.

SCHUMANN, U. † Numerical study of nonlinear effects in homogeneous MHD turbulence.

SCHUMANN, U. 1976 Numerical simulation of the transition from three- to two-dimensional turbulence under a uniform magnetic field. *J. Fluid Mech.* **74**, 31.

SHERCLIFF, J. A. † Alfvén wave power devices.

SHERCLIFF, J. A. 1962 *Theory of Electromagnetic Flow Measurement.* Cambridge University Press.

SHERCLIFF, J. A. 1965 *A Text Book of Magnetohydrodynamics.* Pergamon.

SHERCLIFF, J. A. 1975 Some duct flow problems at high Hartmann number. *Z. angew. Math. Phys.* **26**, 537.

SHERCLIFF, J. A. 1976 Technological Alfvén waves. *Proc. Int. Elect. Engrs*, vol. 123, p. 1035.

SNEYD, A. 1971 Generation of fluid motion in a circular cylinder by an unsteady applied magnetic field. *J. Fluid Mech.* **49**, 817.

SQUIRE, H. B. 1933 On the stability of three-dimensional disturbances of viscous fluid flow between parallel walls. *Proc. Roy. Soc.* A **142**, 641.

SULEM, P. L. & FRISCH, U. † Does MHD turbulence at low magnetic Reynolds number become two-dimensional in the presence of a strong external magnetic field?

Tananaev, A. B. † MHD laminar and turbulent flows in rough ducts.

Tananaev, A. B. 1975 MHD flow in rough tubes. *Magn. Gidro.* **11**, 29.

Temperley, D. J. † MHD flow in a rectangular duct under a uniform transverse magnetic field at high Hartmann number.

Temperley, D. J. & Todd, L. 1971 The effects of wall conductivity in MHD duct flows at high Hartmann number. *Proc. Camb. Phil. Soc.* **69**, 337.

Thatcher, G. 1971 Electromagnetic flowmeters for liquid metals. In *Proc. Int. Conf. on Modern Developments in Flow Measurement*, Harwell, Sept. 1971 (ed. Clayton).

Tir, L. L. 1965 Modelling the motion of molten metal in an induction furnace. *Magn. Gidro.* **1** (4), 120 (MHD **1**, 76).

Tsinober, A. B. 1970 *MHD Flow around Bodies*. Riga: Zinante.

Vivès, Ch. 1974*a* On the coefficient of pressure drag of a cylindrical obstacle in a liquid MHD flow. *C. R. Acad. Sci.* B **278**, 501.

Vivès, Ch. 1974*b* On the coefficient of pressure drag of a non-conducting sphere in a liquid MHD flow. *C. R. Acad. Sci.* B **279**, 203.

Vivès, Ch. 1975 Study of the flow around non-conducting and conducting cylindrical obstacles in the presence of a transverse magnetic field. *C. R. Acad. Sci.* B **280**, 677.

Walker, J. S. † Solitary compression waves in liquid-metal MHD duct flows.

Walker, J. S. & Ludford, G. S. S. 1974*a* MHD flow in conducting circular expansions with strong transverse magnetic fields. *Int. J. Engng Sci.* **12**, 193.

Walker, J. S. & Ludford, G. S. S. 1974*b* MHD flow in insulated circular expansions with strong transverse magnetic fields. *Int. J. Engng Sci.* **12**, 1045.

Walker, J. S., Ludford, G. S. S. & Hunt, J. C. R. 1972 Three-dimensional MHD duct flows with strong transverse magnetic fields. Part 3. Variable-area rectangular ducts with insulating walls. *J. Fluid Mech.* **56**, 121.

Wenger, N. C. 1970 A variational principle for MHD channel flow. *J. Fluid Mech.* **43**, 211.

Wilks, G. † Magnetohydrodynamic free convection about a semi-infinite vertical plate in a strong cross field.

Note. MHD is our abbreviation for the journal *Magnetohydrodynamics*, which is a translation by Faraday Press (New York) of the journal *Magnitnaya Gidrodinamika*, published at Riga in Russian. The latter journal is abbreviated as *Magn. Gidro.*

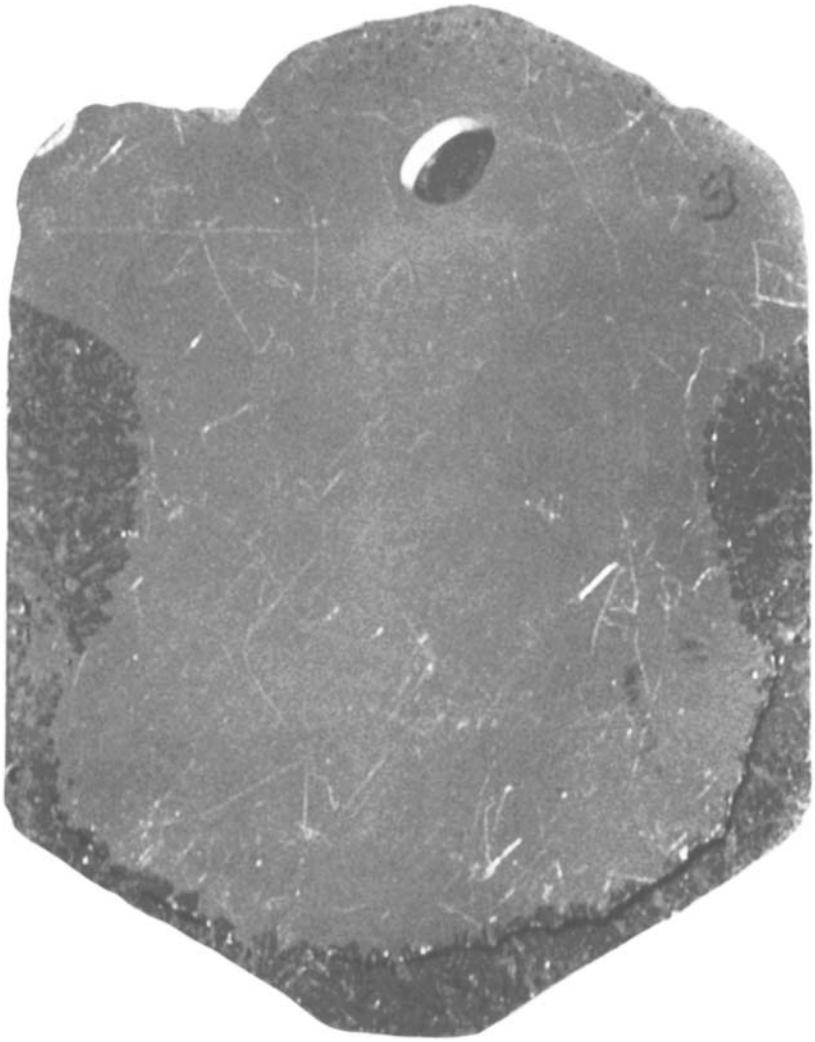

FIGURE 2. MHD interaction in the metallurgical industry. Cross-section of an ingot Al-Si at 30% after solidification. The separation of the phases has been affected by magnetic field at about 0·1 tesla oscillating with a frequency of 2000 Hz. The silicon is at the bottom and at the sides of the ingot.